TRANSITORY AND STEADY ANALYSIS OF GROUNDING STRUCTURES USING THE LN-FDTD METHOD

Rodrigo Melo e Silva de Oliveira
Federal University of Pará
rodrigo@lane.ufpa.br

Carlos Leonidas da S. S. Sobrinho
Federal University of Pará
leondias@ufpa.br

Electrical and Computer Engineering Department - P.O. Box 8619, Zip Code 66075-907, Belém, Pará, Brazil

Abstract—This work presents an overview of the LN-FDTD method (FDTD in local and nonorthogonal coordinate system) to solve Maxwell’s Equations. This method has been used to simulate curved grounding structures. Results are obtained by employing the presented methodology and they are compared to reference equations available in literature.

Index Terms—LN-FDTD, Grounding Systems, Maxwell’s Equations

1 INTRODUCTION

Grounding systems are indispensable for protection of electrical systems and for people safety. In general, grounding systems are composed by metallic electrodes, placed under the ground surface, that are electrically connected to the devices to be electromagnetically protected. The main objective is to take undesirable charges to the ground in a very effective way.

There are many geometries for grounding electrodes, such as vertical or horizontal rods, circular or spherical electrodes, among many others. The performance of the grounding system is highly dependent on these geometries and on the electrical grounding characteristics (conductivity and permittivity).

In a first moment, the grounding systems were present to protect people and buildings from atmospheric discharges and to avoid people to be subjected to electrical discharges from electrical network. Nowadays, the presence of delicate electronic devices in residential, industrial, medical, among other environments increases the importance of grounding systems, as long as they must be protected from undesired currents.

Analysis of grounding grids started with Bewley [1] in 1934, who investigated theoretically and experimentally counterpoise cables. In 1943, vertical grounding rods were theoretically investigated by Bellaschi and Armingtom [2]. Later, Sunde [3] used Maxwell’s Equations to derive expressions for calculating the DC response of various grounding structures.

From the decade of 1980 on, the availability of computers stimulated numerical methods to be employed to analyze grounding structures. Among the used methods, can be emphasized: equivalent circuit models [4], [5], the Method of Moments [6], the Method of Finite Elements [7] and more recently The Finite-Difference Time-Domain Method (FDTD) [8]–[10].

In 2001, Tanabe published the article [9], in which it is shown that the Yee’s algorithm can be employed to analyze rectangular grounding structures. Comparisons between simulated and experimental data are shown and they presented good agreement. Auxiliary electrodes were used to measure voltage and current.

In 2005, Tuma et. al. presented a computational model [10] able to eliminate the electromagnetic coupling between the auxiliary circuits and the grounding structure. The current circuit was replaced by a vertical rod which penetrates the upper absorbing boundary region (U-PML), simulating a natural discharge channel. The voltage circuit was replaced by the calculation of the integral of the electric field.

This work presents a method based on LN-FDTD (Finite-Difference Time Domain Method in local and nonorthogonal coordinate system) to solve Maxwell’s Equations in time domain. The method is employed to simulate grounding structures not coincident to the Cartesian coordinate system, and results are compared to equations available in literature.

2 BASIC THEORY

2.1 The nonorthogonal coordinate system

Consider a general curvilinear region defined by the curves (u₁, u₂, u₃), as illustrated by Fig.1. For this case, the differential lenght vector of \( \vec{r} \) is given by

\[
\vec{dr} = \sum_{l=1}^{3} \frac{\partial r}{\partial u^l} \, du^l = \sum_{l=1}^{3} \vec{\alpha}_l \, du^l,
\]

in which the vectors \( \vec{\alpha}_l \) are called unitary vectors and they form a unitary basis. Of course, the vectors \( \vec{\alpha}_l \) are tangent to the curves \( u^l \) and they can be written as functions of \( x, y \) and \( z \).

A set of three complementary vectors can be defined in such way that each of them is normal to two unitary vectors. The set is given by

\[
\vec{\alpha}_1 = \frac{\vec{\alpha}_2 \times \vec{\alpha}_3}{g}, \quad \vec{\alpha}_2 = \frac{\vec{\alpha}_3 \times \vec{\alpha}_1}{g}, \quad \vec{\alpha}_3 = \frac{\vec{\alpha}_1 \times \vec{\alpha}_2}{g}.
\]

This way, they form the reciprocal basis and are referred as reciprocal vectors. In Equation (2), \( g \) is the determinant of the
metric tensor \([g]\) given by

\[
[g] = \begin{pmatrix}
g_{11} & g_{12} & g_{13} \\
g_{21} & g_{22} & g_{23} \\
g_{31} & g_{32} & g_{33}
\end{pmatrix} = \begin{pmatrix}
\vec{a}_1 \cdot \vec{a}_1 & \vec{a}_1 \cdot \vec{a}_2 & \vec{a}_1 \cdot \vec{a}_3 \\
\vec{a}_2 \cdot \vec{a}_1 & \vec{a}_2 \cdot \vec{a}_2 & \vec{a}_2 \cdot \vec{a}_3 \\
\vec{a}_3 \cdot \vec{a}_1 & \vec{a}_3 \cdot \vec{a}_2 & \vec{a}_3 \cdot \vec{a}_3
\end{pmatrix}.
\]

(3)

After some algebra with Eq.(3), it can be seen that

\[
\sqrt{g} = \vec{a}_1 \cdot (\vec{a}_2 \times \vec{a}_3).
\]

(4)

This means that \(\sqrt{g}\) is the volume of the hexahedron formed by the vectors \(\vec{a}_1, \vec{a}_2\) and \(\vec{a}_3\) (see Fig. 2).

Using the definition provided by Eq.(2), it can be noticed that

\[
\vec{a}_l \cdot \vec{a}_m = \delta_{l,m},
\]

(5)

in which \(\delta_{l,m}\) is the Konecker delta function.

As far as \(g_{lm}\) has been defined for the unitary vectors in Eq.(3), a similar definition can be made for the reciprocal vectors as

\[
g_{lm} = \vec{a}_l \cdot \vec{a}_m.
\]

(6)

With those concepts in mind, it is possible to expand a vector field \(\vec{V}\) using the unitary and reciprocal vectors as (see Eq.(1))

\[
\vec{V} = \sum_{l=1}^{3} v^l \vec{a}_l = \sum_{l=1}^{3} v_l \vec{a}^l,
\]

(7)

in which \(v^l\) and \(v_l\) are called, respectively, the \(l\)'th contravariant component and the \(l\)'th covariant component of \(\vec{V}\).

In order to calculate the \(l\)'th contravariant component of the vector \(\vec{V}\), the dot product \(\vec{V} \cdot \vec{a}_m\) can be calculated. This way, using Equations (5) and (7), one obtains

\[
\vec{V} \cdot \vec{a}_m = \left(\sum_{l=1}^{3} v^l \vec{a}_l\right) \cdot \vec{a}_m = v^m.
\]

(8)

In a similar way, the \(l\)'th covariant component of \(\vec{V}\) can be calculated by

\[
\vec{V} \cdot \vec{a}_l = \left(\sum_{l=1}^{3} v_l \vec{a}^l\right) \cdot \vec{a}_l = v_l.
\]

(9)

A relationship can be obtained to calculate the covariant components from contravariant components and vice-versa. If

\[
\vec{V} \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t}
\]

(13)
Employing the Equations (8) and (2), one obtains
\[
\left(-\mu \frac{\partial \hat{H}}{\partial t}\right) \cdot \hat{a}^1 = \left(\hat{\nabla} \times \hat{E}\right) \cdot \hat{a}^1.
\]

Using the vector identity \((\vec{A} \times \vec{B}) \cdot (\vec{C} \times \vec{D}) = (\vec{A} \cdot \vec{C}) (\vec{B} \cdot \vec{D}) - (\vec{A} \cdot \vec{D}) (\vec{B} \cdot \vec{C})\), it is easy to see that
\[
\left(-\mu \frac{\partial \hat{H}}{\partial t}\right) = \frac{\hat{\nabla} \cdot \hat{a}^2}{\sqrt{\gamma}} \left(\hat{E} \cdot \hat{a}^3\right) - \frac{\hat{\nabla} \cdot \hat{a}^3}{\sqrt{\gamma}} \left(\hat{E} \cdot \hat{a}^2\right).
\]

Observing that \(\hat{\nabla} = \sum_{i=1}^{3} \left(\partial / \partial u^i\right) \hat{a}^i\), from Equation (1) and using Equations (5) and (9) for the right-hand side, one obtains the equation for the first contravariant component of \(\hat{H}\), given by
\[
\frac{\partial h^1}{\partial t} = -\left(\frac{\partial e_3}{\partial u^2} - \frac{\partial e_2}{\partial u^3}\right) / (\mu \sqrt{\gamma}).
\] (15)

Following the same procedure for the two remaining components of \(\hat{H}\) leads to
\[
\frac{\partial h^2}{\partial t} = -\left(\frac{\partial e_1}{\partial u^2} - \frac{\partial e_3}{\partial u^1}\right) / (\mu \sqrt{\gamma}),
\] (16)
and
\[
\frac{\partial h^3}{\partial t} = -\left(\frac{\partial e_2}{\partial u^1} - \frac{\partial e_1}{\partial u^2}\right) / (\mu \sqrt{\gamma}).
\] (17)

For the Electric Field \(\vec{E}\), we have
\[
\frac{\partial e^1}{\partial t} + \sigma e^1 = \frac{\partial h_3}{\partial u^2} - \frac{\partial h_2}{\partial u^1} / \sqrt{\gamma},
\] (18)
\[
\frac{\partial e^2}{\partial t} + \sigma e^2 = \frac{\partial h_1}{\partial u^3} - \frac{\partial h_3}{\partial u^2} / \sqrt{\gamma},
\] (19)
\[
\frac{\partial e^3}{\partial t} + \sigma e^3 = \frac{\partial h_2}{\partial u^1} - \frac{\partial h_1}{\partial u^2} / \sqrt{\gamma}.
\] (20)

Using central differences to approximate Equations (15) and (18), one obtains, respectively, for a structured mesh [11]
\[
\frac{h^1_{i,j,k} - h^1_{i,j,k}}{\Delta u^2} = \frac{e^1_{(i,j,k+1)} - e^1_{(i,j,k)}}{\Delta u^2} + \frac{\Delta t}{\mu \sqrt{\gamma}} \left[ e^1_{(i,j+1,k)} - e^1_{(i,j,k)} - e^1_{(i,j,k+1)} + e^1_{(i,j,k)} \right]
\] (21)
\[
\frac{h^1_{i,j,k} - h^1_{i,j,k}}{\Delta u^3} = e^1_{(i,j,k)} \left(1 - \frac{\Delta t}{\mu \sqrt{\gamma}}\right) + \frac{\Delta t}{(\epsilon + \frac{1}{2} \Delta t \sigma) \sqrt{\gamma}} \left[ h^2_{i,j,k} - h^2_{i,j,k-1} \right] - \frac{\Delta t}{(\epsilon + \frac{1}{2} \Delta t \sigma) \sqrt{\gamma}} \left[ h^2_{i,j,k+1} - h^2_{i,j,k} \right].
\] (22)

It can be noticed that the contravariant components depends on covariant components, which can be calculated using the vector projection (10). It is worth mentioning that \(\Delta u^l (l = 1, 2 \text{ and } 3)\) are considered unitary (equals to one) [11], as long as the cell’s metric information is contained in the term \(\sqrt{\gamma}\).

Mesches for the covariant Electric field must be built in such way that they can properly contour the desired geometry. They are reffered as primary meshes and are formed by primary cells (Fig. 3). As can be seen in Fig. 3, the secondary cells’ corners are positioned at the primary cells’ centers (referred as C in Fig. 3). The secondary meshes points are be calculated from the primary meshes data.
Fig. 3. Covariant components in the Nonorthogonal Yee’s cells: primary for Electric Field and Secondary for Magnetic Field

Fig. 4. Geometry of the analyzed grounding plates

3 RESULTS

3.1 Grounding Plate Electrode

In order to test the presented formulation, it was chosen for simulation the plate grounding system, which geometry is shown by Fig. 4.

According to Yung’s work [12], the steady state value of the $V(t)/I(t)$ rate for this structure is given by Equation (23).

$$R = \frac{13}{100\sigma} \left(a^{-0.818} e^{-0.0093a} \right) \left(1 + e^{-1.427h}\right). \quad (23)$$

For simulating this example, it was necessary to build a 2-D circular mesh, which is shown by Fig.5. In order to obtain the 3-D grid, the 2-D plane of Fig.5 was simply copied to each $u^3$ surface, which are connected by $z$-aligned lines. The generated mesh contains 22 points for representing the disc diameter.

Fig. 6 shows results obtained for the $V(t)/I(t)$ ratio for a plate with $2a = 5.5$m, located 0.5m under the ground surface. The used ground parameters are $\sigma = 2.28$mS/m, $\epsilon_r = 50$ and $\mu = \mu_0$.

For this case, Eq. (23) provides, $32.69\Omega$ and the LN-FDTD method resulted in $33.42\Omega$, as can be seen by Fig. 6, what is a very close numerical approximation. Other cases were tested and similar approximations were obtained.

It is worth to mention that meshes with more than 22 points were tested for the present problem. However, the obtained results where not very different from those presented here. This means that the method’s convergence is assured.

The methodology presented in [10] for calculating the transient voltage and current has been employed (taking into account the basis of the local coordinate system).

3.2 Simulation of a Semispherical Electrode

In order to test a more complex example, the semispherical electrode illustrated by Fig.7 was modeled. A slice of the
generated computational mesh is illustrated by Fig.8, with half of the actual resolution.

As it is well known, the steady value of \( R = V(t)/I(t) \), for \( h = 0 \), is given by

\[
R_{\text{steady}} = \frac{1}{2\pi a \sigma}.
\]  

(24)

Fig.9 shows the obtained relation \( V(t)/I(t) \) when \( \sigma = 2.28\text{mS/m} \), \( \epsilon_r = 10 \), \( a = 6\text{m} \). For this case, (24) provides 11.63\( \Omega \), while, at 0.5\( \mu \text{s} \), it was obtained 12.55\( \Omega \) via simulation (Fig.9), that is, less than 1\( \Omega \) of deviation from the analytical solution. It is also possible to observe in Fig.9 that before 0.1\( \mu \text{s} \), current reflects at the semispherical electrode \( (h = 0) \), promoting the increase of the relation \( V(t)/I(t) \) present in the curve transitory. After that, the curve tends to converge to the steady solution. The methodology presented in [10] for obtaining the transient voltage has been employed.

It should be mentioned that when the mesh is structured, it is hard to keep all the cells with nearly the same volumes and also to avoid great inclinations of the edges of the cells. Due this motive, the late time stability problems mentioned by Gedney at [13] were observed. Therefore, the simulation was limited to 0.5\( \mu \text{s} \). For this kind of problem, unstructured meshes seem to be more adequate for performing long time simulations.

Figure 10 shows the distribution of the component \( e_3 \) at a plane crossing the middle of the semispherical electrode. It is possible to observe the proper absorption of the waves by the virtual anechoic chamber modeled using the presented formulation. It is also possible to observe the field contouring
The obtained relation $V(t)/I(t)$ for the Semispherical Electrode (Fig. 9).

The $\varepsilon_3$ component of the electric field (dB) at the beginning of the transient period, showing the absorption provided by the developed UPML formulation (Fig. 10).

the spherical electrode and the different propagation speeds in free space and in the ground (it is possible to observe the field continuity at the air-ground interface).

4 Conclusions

A methodology for analyzing grounding systems was developed based on the numerical solution of Maxwell’s Equations in Local Non-orthogonal coordinate system. The solution is obtained by the application of the Local-Nonorthogonal Finite-Difference Time-Domain Method (LN-FDTD). The method was truncated by a special implementation of the UPML (Uniaxial Perfectly Matched Layers) in that coordinate system for conductive media.

It is worth to mention that LN-FDTD provides identical results when compared to FDTD when orthogonal structures were tested. However, when FDTD is used to approximate, for example, the plate electrode, it is necessary to use a four times denser mesh in order to produce results close to the presented here, specially during the transitory period.

The main advantage of using this formulation is, therefore, the possibility to simulate structures not coincident to the Cartesian system and obtain accurate results in smaller processing time periods when compared to the orthogonal FDTD method. This work shows results obtained for a plate and for a semispherical electrode, which agree very well with equations available in literature.

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6 References